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CONFINEMENT AND THE PION NUCLEON SIGMA TERM

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ABSTRACT

Gribov's theory of confinement offers a simple explanation of the value of the pion nucleon sigma term. There is no need to invoke a large strange quark component in the nucleon.

It has long been thought that there is a discrepancy between the value of the pion nucleon sigma term $\sigma_{\pi N}$ measured in π N scattering and the theoretical prediction of $\sigma_{\pi N}$ from hadron spectroscopy (see eg. [1] and references therein). In this paper I explain why this "discrepancy" is a natural consequence of Gribov's mechanism for confinement [2, 3].

It will be helpful to first review the theory of $\sigma_{\pi N}$ in the usual theory of pions and chiral symmetry [4]. The pion nucleon sigma term is a measure of chiral symmetry breaking in the nucleon. The QCD Lagrangian exhibits exact chiral symmetry for massless quarks. Since there are no parity doublets in the hadron spectrum we know that the chiral symmetry must be spontaneously broken, whence Goldstone's theorem tells us to expect a zero mass boson. In the real world the light quarks have a small mass, which breaks the exact chiral symmetry. We use $H_m = \hat{m}\overline{q}q$ to denote the chiral symmetry breaking term in the QCD Lagrangian where $\overline{q}q = \overline{u}u + \overline{d}d$ and $\hat{m}$ is the mean light "current-quark" mass. If we assume that hadronic physics changes continuously (with no phase transition) as we vary the light quark mass from zero to $\hat{m}$ then the chiral Goldstone state acquires a small mass. It is identified with the physical pion. The theory of chiral symmetry gives a relation between the value of $\hat{m}$ and the pion mass, viz.

$$m_\pi^2 = \hat{m}\left(\frac{-<vac|\overline{q}q|vac>}{f_\pi^2}\right) \tag{1}$$

which we shall need later in our discussion. The sigma term is formally defined as

$$\sigma_{\pi N} = \frac{1}{3}\sum_{i=1}^{3} <N|\left[Q_5^i, \left[Q_5^i, H_m\right]\right]|N> \tag{2}$$

where Q_5^i is the axial charge. After we use the QCD equations of motion to evaluate the commutators in equ.(2) $\sigma_{\pi N}$ becomes

The value of $\sigma_{\pi N}$ is measured in π N scattering to be $\sigma_{\pi N} \simeq 45$MeV [5]. In renormalised QCD $[m\overline{q}q]$ is scale invariant. For this reason it is commonly assumed that vacuum polarisation in QCD does not play an important role in the physics of chiral symmetry. Using the QCD sum-rule determination of $- < vac|\overline{q}q|vac >$ one finds that $\hat{m} = 6$MeV at a scale $\mu^2 = 1\text{GeV}^2$ [6].

The determination of $\sigma_{\pi N}$ from hadron spectroscopy goes as follows. The mass degeneracy in the baryon octet is broken by the finite quark masses. If we make a leading order analytic (linear) expansion in the quark mass and assume that there is a negligible strange quark component $< N|\overline{s}s|N >= 0$ in the nucleon, then it is easy to show that [6]

$$\sigma_{\pi N}^{th} = \frac{3(M_{\Xi} - M_{\Lambda})}{(\frac{m_s}{m_q} - 1)} \tag{4}$$

where m_q and m_s are the running light quark and strange quark masses respectively. If we identify $m_q = \hat{m}$ (whence $\frac{m_s}{m_q} = 25$), then we find the familiar prediction of hadron spectroscopy $\sigma_{\pi N}^{th} = 25$MeV. The difference between this theoretical prediction and the value of $\sigma_{\pi N}$ measured in π N scattering is the sigma term "discrepancy". It has led to suggestions (see [1]) that there might be a large strange quark component $< N|\overline{s}s|N >$ in the nucleon. If this were true then a significant fraction of the nucleon's mass would be due to strange quarks – in contradiction with the quark model.

The derivation of equs.(1-4) assumed that hadronic physics changes continuously (with no phase transition) as we take $m_q \to 0$. At this point we have to be careful. Even in QED we know that the theory of the electron differs from the theory with a zero mass gap. The Born level cross section for e^+e^- production when a hard (large Q^2) transverse photon scatters from a soft (small $-p^2$) longitudinal photon is non-vanishing when we let $-p^2 \to 0$ in QED with a zero mass gap [7]. This is in contrast to the familiar physical situation where the electron has a finite mass and this cross-section vanishes as $-p^2 \to 0$. As we take $m_e \to 0$ the vacuum in QED becomes strongly polarised: the perturbation theory expression for the vacuum polarisation $\Pi(q^2)$ diverges logarithmically. This suggests that the vacuum state for QED with a zero mass gap is not perturbative: it exists in a different phase of the theory [8]. In QCD there is every reason to expect vacuum polarisation to play an important role in the physics of light quarks since the QCD dynamics at strong coupling must spontaneously break the chiral symmetry of the classical theory.

We now present a simple explanation of the sigma term "discrepancy" in terms of Gribov's theory of confinement [2,3]. Gribov's idea is that QCD becomes super-critical at some finite $\alpha_s^c \sim 0.6$, at which point the energy level of a quark in a background colour field falls below the Fermi surface of the perturbative vacuum. The quark then becomes a resonance and is not seen as a free particle.

The Gribov theory is obtained via the simultaneous solution of the Schwinger-Dyson equation for the quark propagator and the Bethe-Salpeter equation for meson bound states. At a critical coupling $\alpha_s^c \sim 0.6$ the theory exhibits a rich sequence of phase transitions. There appear multiple solutions to the quark propagator equation which correspond to new states in the light quark vacuum. These new vacuum states correspond to quasi-particle excitations with both positive and negative masses [2]. Each phase transition leading to new states in the vacuum is characterised by a critical mass m_P^c. Provided that the running quark mass m_P at the critical scale λ (where $\alpha_s = \alpha_s^c$) is less than the critical mass $m_P^c \ll \lambda$ one finds that the solution of the Schwinger-Dyson equation in the new vacuum states matches onto the solution of the perturbative renormalisation group equation, which describes the physics at small coupling. If this condition is satisfied then the new vacuum states yield physical excitations in the theory. (For technical details and a derivation of these results see [2].)

Each transition is characterised by a pseudo-scalar Goldstone bound state. (The appearance of the new vacuum states spontaneously breaks the chiral symmetry.) The wavefunctions of the physical Goldstone states are found by perturbing the solution of the Bethe-Salpeter equation about the critical mass m_P^c. The mass of the Goldstone state associated with any given phase transition is determined by m_P and the value of $m_P^c(\lambda)$ for that transition. One finds that the Goldstone meson mass is [2]

spectrum tells us that the light quark mass lies somewhere between the critical mass for the first and the second vacuum transitions.

We compare equs. (5) and (1), whence it follows that the "current-quark" mass in pion physics (and in particular equ.(3)) is

$$\hat{m} = m_P^c - m_P \tag{6}$$

rather than m_P. The QCD dynamics which lead to the chiral phase transition tell us that chiral perturbation theory (in pion physics) is really an expansion about $(m_P^c - m_P) = 0$ rather than about $m_P = 0$. (The usual formulation of chiral symmetry *assumes* that $m_P^c = 0$, which need not be the case.) As we decrease $m_P \to 0$ the mass of the *physical* pion increases in the Gribov theory. When m_P coincides with the critical mass for the nth transition ($n \geq 2$) a new Goldstone state appears in the hadron spectrum with zero mass. The mass of this state increases as we decrease m_P further. At $m_P = 0$ one finds an infinite number of pseudo-scalar Goldstone states with small masses $\mu^2_{\pi,n}$, where $\mu^2_{\pi,n} \geq \mu^2_{\pi,n+1} \to 0$ as $n \to \infty$ [2].

If we compare equs. (1) and (6), it follows that

$$\kappa^2 = \left[\frac{- < vac|\overline{q}q|vac >}{f_\pi^2}\right]_{\mu^2=\lambda} \tag{7}$$

in the usual theory of pion physics [4]. Using the scale invariance of $[m\overline{q}q]$, chiral perturbation theory tells us that

$$m_P^c - m_P = 6MeV \tag{8}$$

after we evolve to $\mu^2 = 1\text{GeV}^2$.

Whilst the "current-quark" mass in pion physics is $\hat{m} = m_P^c - m_P$ (equ.(6)), the mass in the renormalised QCD Hamiltonian is (of course) the running quark mass m_P. The mass degeneracy of physical non-Goldstone states (like the baryon octet) is broken by the finite values of the running masses m_P. The baryon mass M_B is sensitive to two effects as we vary the light quark mass between zero and the critical mass m_P^c for the first transition. Firstly, the mass of the constituent quark quasi-particle excitation increases with m_P. When m_P is less than the critical mass for the nth transition the baryon mass also receives "pion" corrections associated with the nth Goldstone state. The leading "pion" correction to M_B in improved chiral perturbation theory is proportional to $\mu_\pi^2 \ln \mu_\pi^2$ [6], where μ_π^2 is proportional to $(m_P^c - m_P)$ in the Gribov theory. When m_P is increased above m_P^c these Goldstone states condense in the vacuum leading to a further increase in the mass of the constituent quark [2]. The quark mass which appears in the linear mass expansion (which does not include "pion" corrections) for the baryon octet is the running mass m_P. This means that m_P is the light quark mass in equ.(4). The phase transitions in the light quark vacuum mean that the Gribov theory anticipates a "discrepancy" between the value of $\sigma_{\pi N}$ which is measured in π N scattering and the value of $\sigma_{\pi N}^{th}$ which is extracted from baryon spectroscopy. This "discrepancy" would remain even if the effect of "pion" corrections to M_B were included. There is no theoretical need to introduce a large strange quark matrix element in the nucleon in order to explain the $\sigma_{\pi N}$ data.

We can use equ.(4) to estimate the value of m_P^c at 1 GeV^2. The old spectroscopy prediction of the sigma term used $m_q = (m_P^c - m_P)$ instead of $m_q = m_P$. We substitute the measured value $\sigma_{\pi N} = 45$MeV into the linear mass formula

$$\sigma_{\pi N}(\frac{m_s}{m_q} - 1) = constant \tag{9}$$

and use the result that the strange "current-quark" mass $m_s \simeq m_{s,P}$ to obtain $m_q \simeq 10.5$MeV at a scale $\mu^2 = 1\text{GeV}^2$. It follows that the value of the critical mass for the chiral phase transition is $m_P^c \simeq 16.5$MeV after evolution to $\mu^2 = 1\text{GeV}^2$. A detailed analysis of pion and kaon physics in the Gribov theory will be given in the next paper [9].

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